*Using Puzzles to Teach Problem Solving*

# Teacher's Guide to IZZI and IZZI 2

## Benefits

IZZI and IZZI 2 are a pattern matching games that are an exciting way to teach elements of

— problem solving
— pattern recognition
— spatial thinking
— combinatorial reasoning

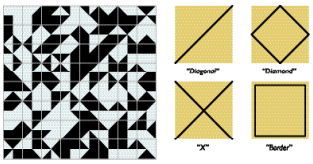
The activities in this guide give students several new ways of playing the games, both as solitaire and as multiplayer games, and explain some of the math behind the game.

## About IZZI

IZZI includes 64 square tiles, each divided into a different pattern of black and white regions. The goal is to arrange the tiles into an 8 by 8 square (or other pattern) so the edges of touching tiles match: black touches black and white touches white. Notice how the edges of the loose pieces below match the edges of the pieces they will touch.

IZZI has thousands solutions. Notice how the solution shown below satisfied the additional requirement of forming a "Diagonal" across it. For a harder challenge students can also try solving IZZI and creating one of the other patterns below. "Border" is the hardest pattern.

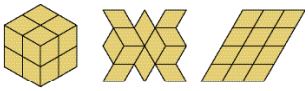
In the course of playing IZZI students practice recognizing and mentally rotating spatial patterns. The visual patterns suggest art projects. The IZZI tiles are also a good way to introduce the basics of combinatoriial reasoning.




## About IZZI 2

IZZI 2 includes 12 diamond-shaped tiles, each divided into a different pattern of red, blue, green and yellow triangles. The goal is to arrange the 12 tiles in a hexagon (or other shape) so edges of touching tiles match; red touches red, blue touches blue, green touches green and yellow touches yellow. Notice how the edges of the loose pieces below match the edges of the pieces they will touch.

Like the original IZZI, IZZI 2 encourages students to recognize and mentally rotate spatial patterns, and serves to introduce the basics of combinatorial reasoning. In addition to forming a hexagon, students can also try forming any of 38 other patterns printed in an enclosed booklet, with the same condition that edges of touching pieces must have the same color.



The differences between IZZI and IZZI 2 are that IZZI 2 has fewer pieces, is based on a triangular rather than a square grid, and involves color instead of black and white. Mathematically the difference is that colors do not repeat within an IZZI 2 tile, whereas colors (black and white) often repeat within an IZZI tile, leading to different strategies for counting the number of possible tiles.

IZZI and IZZI 2 were invented by New York City graphic designer Frank Nichols.

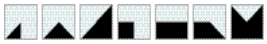
# INTRODUCING IZZI

Play the game, reflect on experience

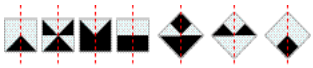
**Become familiar with the pieces** .

— Lay out the IZZI tiles on a table. Count the tiles. Are any two the same? (Answer: 64 tiles, all different.)

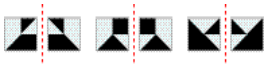
— What sorts of black shapes appear on the tiles? Can you find triangles in three sizes? Can you find a square? A rectangle that is not square? A trapezoid? A five-sided polygon? Can you find the same shapes in white?



— Find tiles that have mirror symmetry, such as:

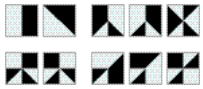


— Find tiles that form mirror images pairs. such as:

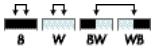


— Find tiles that have rotational symmetry, meaning that the tile looks the same if rotated 180°. There are only 4 such tiles in IZZI. Other rotationally symmetric tiles are possible, but they are not part of IZZI. Speaking of symmetry, what is special about the word "IZZI"? What other words have this property?

— Find tiles in which the black areas are the same shapes as the white areas. There are 10 such tiles. Other such tiles are possible, but they are not part of IZZI.



— Look at just one edge of a tile. How many different possible edge patterns are there? Which types can border which other types? Answer: four types. B=black, W=white, BW=black-white and WB=white-black. B borders B, W borders W, and BW borders WB.

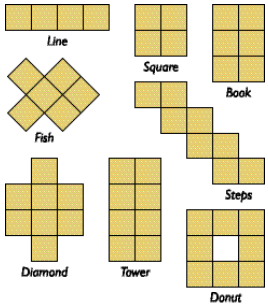



**Mini-IZZI** (based on an idea by Dale Seymour)

The full game of IZZI takes time and desk space. Here is a smaller faster game that up to 8 students can play at the same time. Unlike full IZZI, Mini-IZZI works fine even if you lose a few tiles.

Each student chooses 8 tiles at random. Make each of the patterns shown below using tiles from your set of 8. As in IZZI, tiles must match along edges. To play competitively, call out the name of a pattern when you complete it, and score a point if you made that pattern first.

Question for advanced students: can you pick a set of 8 tiles that cannot make some of these patterns? Is there a set of 8 tiles that cannot make any of these patterns?




**Full IZZI** . Play the full game of IZZI by assembling all 64 pieces into an 8 by 8 square. Colors must match along edges. Students can work individually or in small groups.

**Reflect on experience** . As with all manipulatives, students will get more out of IZZI if they take time to reflect on their experiences.

— Write about your thought processes. What problem solving strategies did you try? Which worked well and which did not?

— What did you do when you got stuck? Explain through words or diagram the strategies you tried in solving a particular puzzle?

— What advice would you give to other students trying to solve IZZI puzzles?

# INTRODUCING IZZI 2

Play the game, reflect on experience

**Become familiar with the pieces** . Lay out the IZZI 2 pieces on a table.

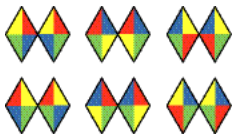
— How many tiles are there? How many different colors are there? Are any two tiles exactly alike? Answer: no.

— Mirror images. Find two tiles that are mirror images of each other. Can you divide all 12 tiles into 6 pairs of mirror images? Answer:

— Same in one place. Choose one tile and turn it so one of the sharp angles points straight up. Look at the color in the upper left quadrant. How many other pieces can you find with the same color in that quadrant? Answer: 5.

What do the remaining six pieces have in common? Answer: that same color in the upper right quadrant.

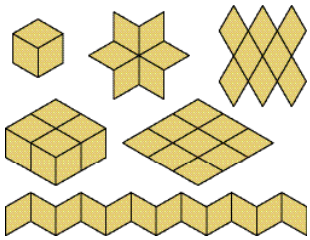
— Same in two places. Choose one tile and turn it so one of the sharp angles points straight up. Look at the colors in the upper half. How many other pieces can you find with the same color in that half? Answer: just 1, with the bottom two colors reversed.. Can put all 12 tiles in pairs so both tiles in each pair have the same top half? Here is one solution:



— Same in three places. Are there two pieces that have the same colors in three places? Why or why not? Answer: no, because then the two pieces would have all four quadrants colored exactly the same.

**Mini-IZZI 2.** Arrange IZZI 2 tiles to make each of the patterns below. Tiles may touch only if colors match along edges.

Question for advanced students: For each pattern, can you choose a set of tiles with the proper number of tiles that cannot be assembled to make that pattern?



**Full IZZI 2** . Have students build the 39 patterns in the booklet either individually or with a partner. Record which patterns you have made.

**Reflect on experience** . As with all manipulatives, students will get more out of IZZI if they take time to reflect on their experiences.

— What types of patterns are harder to build than others, and why?

— Write about your thought processes. What problem solving strategies did you try? Which worked well and which did not?

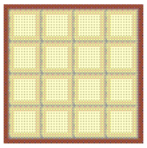
— What did you do when you got stuck? Explain through words or diagram the strategies you tried in solving a particular puzzle?

— What advice would you give to other students trying to solve IZZI puzzles?

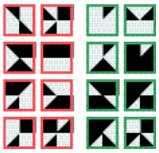
# ONE-ON-ONE IZZI

Game for 2 students

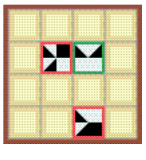
The full game of IZZI takes a long time and is for only one player. Here is a quicker version of IZZI for two players that is highly competitive. The same game also works for IZZI 2.



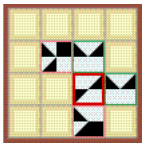
**Board.** Print out the board shown on page 5, or draw it on paper. For a longer game, try a 5 by 5 or 6 by 6 board.



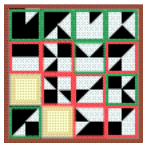
**Tiles.** Each player chooses 8 tiles at random. (Increase the number of tiles per player to make the game easier.) Lay out tiles where both players can see them. For clarity, player 1 tiles are outlined here in red, and player 2 tiles are outlined in green.



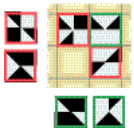
**Moves** . To begin the game, player 1 places a tile on any empty space. Then player 2 places a tile on any empty space. Player then take turns placing tiles on the board. Tiles that touch must match along edges. Here is the game after player 1's second move.



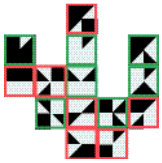
**Scoring** . Each tiles scores 0 to 4 points equal to the number of tiles it borders. For instance, the move highlighted here in dark red scores 3 points because it borders three other tiles.



**The game ends** when the board is full or neither player can move. The player with the most points wins. In this game each player is left with one piece that cannot be placed on the board.

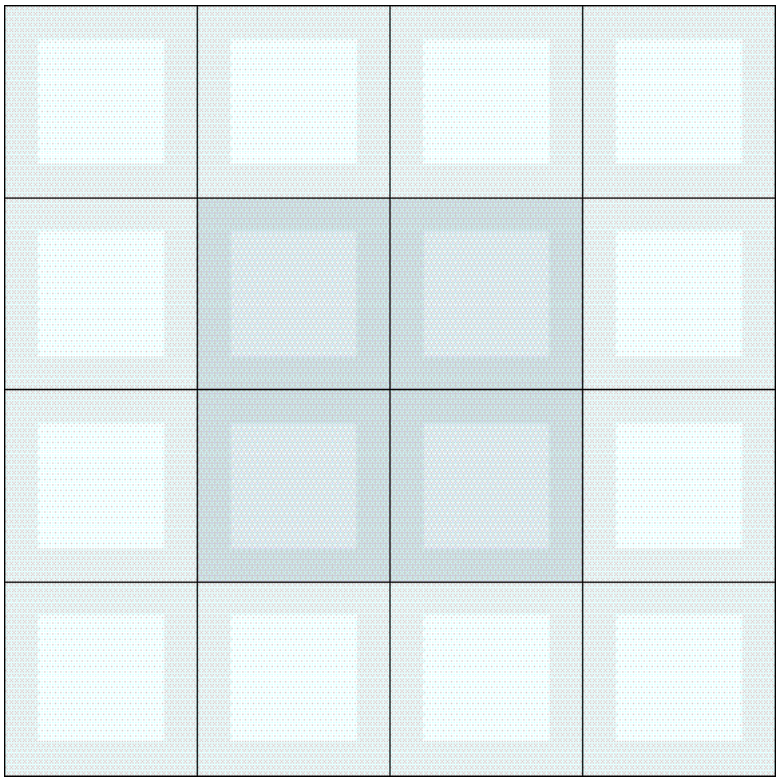


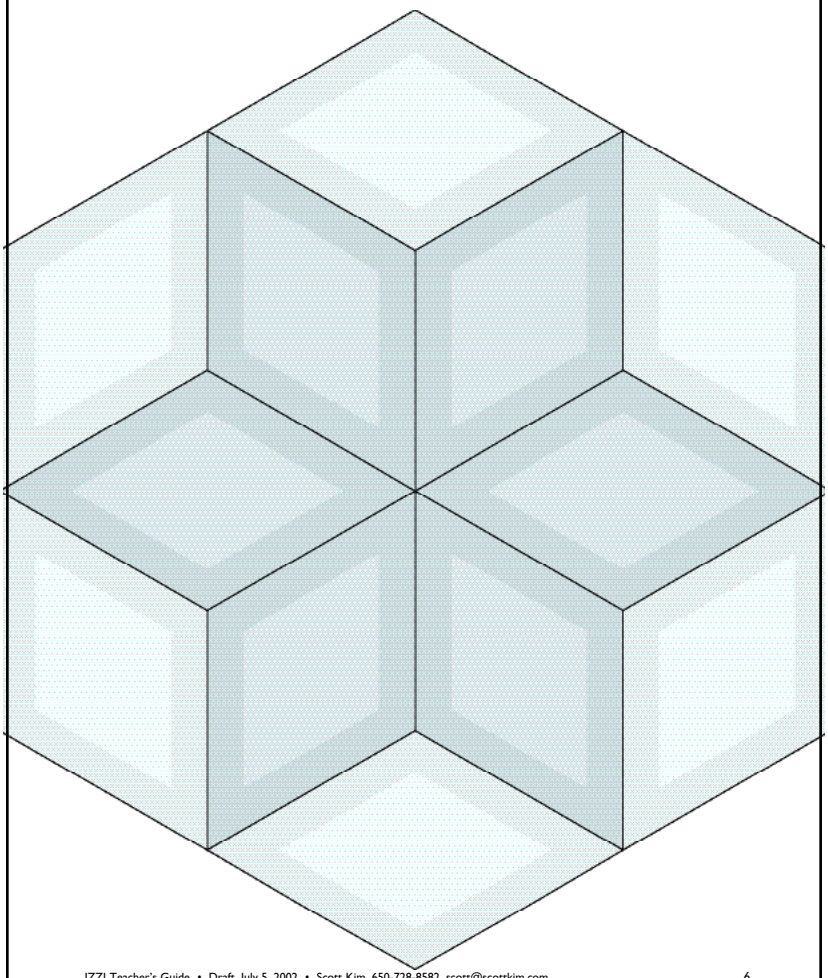
**Toward the end** of the game it is common to get stuck with no place to move. A good strategy is to create positions where you can move but your opponent cannot. For instance, in the position at left, red can play a piece in the empty square, but green cannot.



**Freeform variation** : play without a board, with the requirement that each piece (except the first) must border another piece. In this variation it is much harder to get stuck without a move Tip: set up multipoint moves that you can make but your opponent cannot.

**The same game works for IZZI 2** . Print out the board on page 6, or draw it on paper. Each player gets 12 tiles. Players take turns placing tiles on the board. Each tile scores points equal to the number of tiles it borders. Try the freeform variation too.





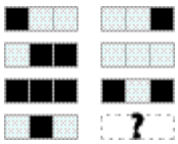
# WHAT'S MISSING FROM IZZI?

An investigation in systematic thinking

**There are 64 tiles in IZZI. Each tile has a different black and white pattern. A few of the possible patterns are missing. What's missing?**

This simple question challenges students to organize information into a systematic order — a basic problem solving skill. You can use this investigation with the whole class, or give it to small groups to solve on their own. You will need a complete set of 64 IZZI tiles for this exercise. Although this exercise is challenging and touches on sophisticated ideas, no algebra is required.

**What's missing?** Before we get to IZZI tiles, here is a simpler warm-up question. Ask students what's missing from the set of rectangular tiles shown below. The order of the tiles is not important. Then ask students to explain their reasoning.




**The rules** . To identify the missing tiles, we must first describe what patterns belong in IZZI. Ask students to describe what all the IZZI tiles have in comon. List the descriptions on the board. For instance, students say:

— All tiles are square.

— All tiles are black and white.

— All shapes on the tiles have straight edges, no curves.

Then ask students to see if they can draw tiles that follow the rules, but should not be allowed as IZZI tiles. For instance, these tiles obey the three rules above but should probably not be counted as IZZI tiles.

Now challenge students to add rules to their list until no tiles except IZZI tiles are allowed. Students will disagree about the interpretations of rules; let them work out resolutions. For instance, the final rules might be:

— All tiles are squares of the same size.

— Each tile is divided into eight triangles by lines that connecting opposite corners and midpoints of edges.

— Each triangle can be colored either black or white.

— Two tiles are considered the same if one can be turned to match the other.

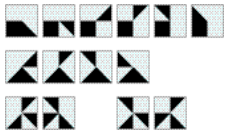
**Listing tiles in order.** To help them find the missing tiles, ask students to figure out a way to sort all 64 tiles into an order that is certain to list every pattern just once.

A good approach is to divide and conquer. First find a way to systematically divide the 64 tiles into smaller groups. You can let students devise their own approach, or tell them the following method.

A good way to divide and conquer is to count the number of black triangles in a tile (or, equivalently, white triangles). There are eight triangles, so there should be 9 groups corresponding to 0 through 8 black triangles. Have students work together to sort the IZZI tiles into groups.

Students will find there are no tiles with 0 or 8 black triangles. The sizes of all the groups are listed at right. Ask students to check for missing patterns by putting tiles in each group into an order. Shown below is one way to order the 3-triangle tiles. Can you explain the logic behind this order? How can you be sure that this sequence includes all possible tiles with three black triangles?

| | |
|---|---|
| **0** | (0 triangle) |
| **2** | (1 triangle) |
| **8** | (2 triangles) |
| **14** | (3 triangles) |
| **16** | (4 triangles) |
| **14** | (5 triangles) |
| **8** | (6 triangles) |
| **2** | (7 triangles) |
| **0** | (8 triangles) |



The 4-triangle group is the hardest to sort. There are four missing 4-triangle tiles, and all other groups are complete. Altogether, there are 6 missing tiles, shown below:

**Why omit those 6 tiles?** One final discussion question. How do you think IZZI's creator chose which 6 tiles to omit from IZZI? What other choices could he have made? If you were in charge of IZZI, what would you have done?

# Counting Combinations

## The algebra behind IZZI

Here is a more sophisticated treatment of counting possible IZZI tiles. Familiarity with algebra is recommended. Counting problems like this are a basic part of combinatorics and discrete mathematics.

**Counting square tiles** . Coloring each of the eight triangles in an IZZI tile either black or white is like assigning each of the places in an 8-digit binary number either 1 or 0. How many different eight digit binary numbers exist? The answer is $2*2*2*2*2*2*2*2 = 2^8 = 256$, since each digit choice multiplies the number of possibilities by 2. Similarly, there are $2^4 = 16$ different 4-digit binary numbers, and $2^{10} = 1024$ ways to hold up fingers on both hands.

So why aren't there 256 IZZI tiles?

The answer is rotation. We don't count two tiles as different if one is the same as the other rotated, and a square can be rotated to 4 different positions. (We do not allow rotations that turn back to front.) That divides the number of possible tiles by 4. For instance, the following four tiles should count as just one.

So why aren't there exactly 256/4 = 64 possible IZZI tiles?

The answer is symmetry. The following tiles have 4-fold rotational symmetry, which means they look the same when rotated by 90°:

When 256 is divided by 4, these 4 tiles are counted as just 1, when they should be counted as 4. So we need to add 3 to the total number of tiles.

The following tiles have 2-fold rotational symmetry, which means they look the same when rotated by 180°:

When 256 is divided by 4, these 6 tiles and their 90° rotations (12 tiles total) are counted as just 3, when they should be counted as 6. So we need to add 3 again to the number of tiles. Altogether, 64+3+3 = 70 possible tiles.

Expressed as an algebraic formula, the total number of possible IZZI tiles with n=4 sides is

(number of black and white colorings) +
(correction for tiles with n-fold rotational symmetry) +
(correction for tiles with 2-fold rotational symmetry)
$= (4^n / n) +$
$[(n/1) - (n/n)] +$
$[(n*(n-1))/2 - (n*(n-1))/n]$

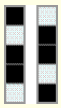
**Beyond squares** . How many triangular IZZI ti' are possible? That is, how many different ways are there to color the triangles in the template at right black or white, counting two tiles as the same if one can be turned to match the other?

Answer: $(4^n + (n-1)*4)/n = (64+8)/3 = 24$

How many pentagonal IZZI tiles are possible? That is, how many different ways are there to color the triangles in the template at right black or white, counting two tiles as the same if one can be turned to match the other?

Answer: $(4^n + (n-1)*4)/n = (1024+16)/5 = 208$

How many different strips of five squares colored black or white are possible? Count two strips as the same if one can be rotated 180° to match the other. For instance, here are two equivalent 5-strips.



Answer: $2^5 + (2^5-2^3)/2 = 32-12 = 20$

**Beyond black and white** . Do the IZZI 2 tiles include all possible arrangements of 4 colors? If not, what's missing?
Answer: yes, IZZI 2 includes every possible combination.

If IZZI 2 tiles were squares instead of diamonds, how many different tiles would be possible? Answer: 6.

If IZZI 2 tiles allowed any of the four colors to repeat within a tile, how many different tiles would be possible?
Answer: $(4^4/2) + (4^2/2) = 128 + 8 = 136$

# Create Your Own

Open-ended creative activities

Problem posing is an important part of problem solving, since in real life there is no teacher to tell you which problems to solve. These activities that involve students in creating their own IZZI and IZZI 2 challenges.

**Create IZZI line patterns.** You can invent your own line patterns similar to the ones printed on the IZZI box. Using the grid on page 10, draw a pattern of lines connecting black dots horizontally, vertically or diagonally. Then see if you can make the line pattern using all 64 IZZI tiles. Of course colors must match along edges.

Note: some line patterns that use a lot of lines are impossible to solve.

**Create IZZI 2 tile patterns** . You can draw your own layouts for the 12 pieces, similiar to the ones printed in the IZZI 2 booklet. Using the grid on page 11, draw a shape made of 12 diamonds. Then see if you can make the shape using all 12 IZZI 2 pieces. Of course colors must match along edges.

Unsolved problem: Are there any patterns that are impossible to make with the 12 IZZI 2 tiles?

**Create new games** . Invent new ways to play games with the IZZI and IZZI 2 tiles. Here are some ideas to get you started.

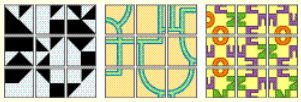
- Play around with the tiles and see what ways of playing they suggest.
- Think of a game you like and see if you can make up a game similar to it using the IZZI or IZZI 2 tiles.
- Make up a game that resembles a familiar card game, such as Fish, using the IZZI tiles.
- Invent a game with the IZZI 2 tiles that involves dice?
- Play a game on a chessboard in which the pieces that you move from square to square are IZZI tiles, and the pattern on the tile determines how the piece moves.
- Come up with a logical deduction game similar to Mastermind using the IZZI or IZZI 2 tiles.
- Create a game that is played on a 6 by 6 board made up of IZZI tiles.

## Edge Matching Puzzles

IZZI is an example of an edge-matching puzzle — a puzzle in which the goal is to arrange a set of tiles so the patterns printed on them match along edges. The first edge matching puzzles with square tiles was patented in 1890. Dominoes, which are related to edge-matching puzzles, are much older.

The most common sort of edge-matching puzzle, by far, has 9 square pieces to be arranged in a 3 by 3 square. Although such puzzles look easy, they are frequently quite difficult. Often the patterns that must match at edges are little pictures that follow a theme, such as animals or people.

Here are three 3 by 3 edge-matching puzzles with different pictures, but identical logic. To see the similarities, identify the four types of edges in each puzzle.

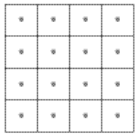


There are many other types of edge-matching puzzles. Triazzle has triangular tiles. Trax is a game with square tiles that connect to form continuous roads. Some variations disallow rotating tiles, and other variations use notched edges instead of printed patterns to indicate which edges can go together.

**Create new tiles** . Make up your own set of tiles, similar to IZZI or IZZI 2. Here are some things to try.

- **Other colors** . What would a 3-color IZZI look like? What if colors in IZZI 2 could repeat within a tile?
- **Other patterns** . Instead of triangles radiating out from the center, what other patterns could you draw on each tile? See examples above for ideas.
- **Other tile shapes** . Explore other geometric shapes, such as rectangles or hexagons. What can you do with pattern blocks or tangrams?
- **Make a set.** Draw your tiles with the aid of a computer drawing program. Print your tiles on heavy paper and cut them out. Put an identifying number on each tile so you can record solutions easily.
- **IZZI 3** . Suppose you were commissioned to design the next sequel, IZZI 3. What would you do? The new game should clearly resemble IZZI and IZZI 2, yet introduce a fresh new twist.

Name: _____



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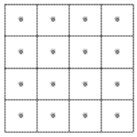

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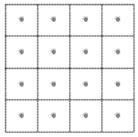

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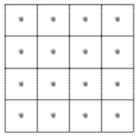

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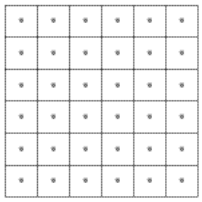

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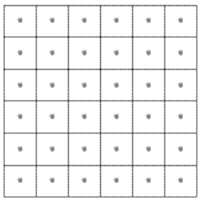

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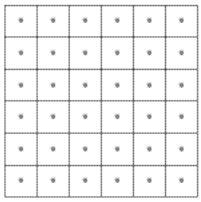

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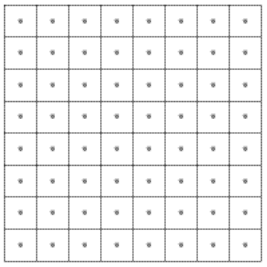

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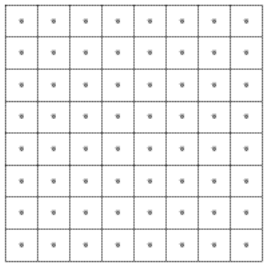

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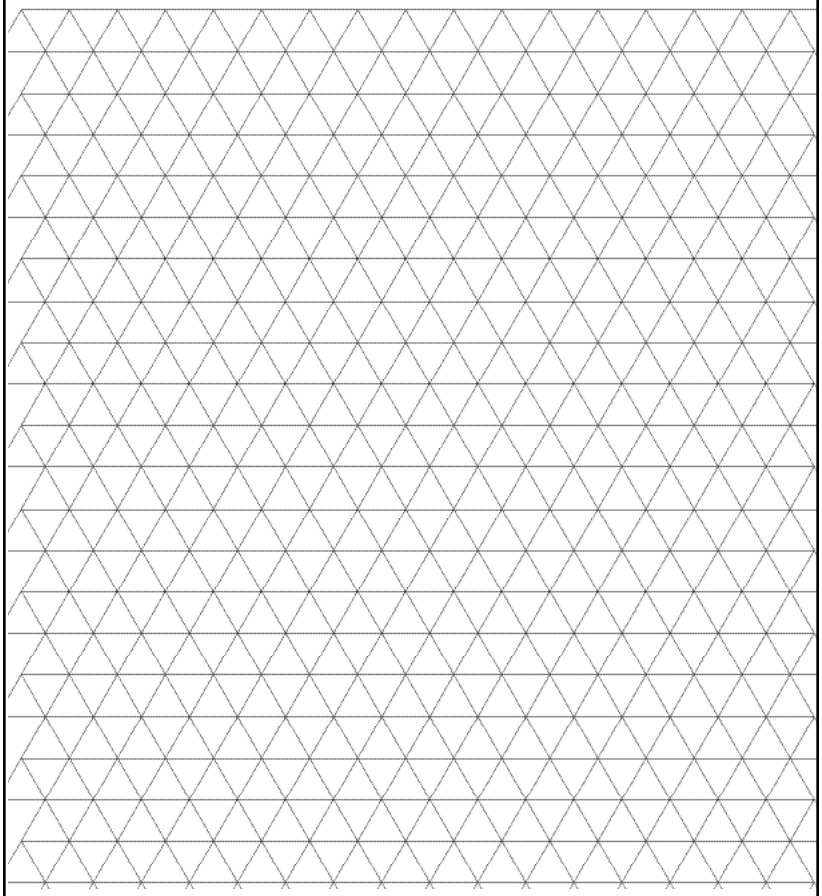

# LEARNING FROM IZZI

Big open-ended discussion questions

**Visual thinking** . What are some of the visual imagination skills that IZZI and IZZI 2 exercise? Where else might these skills be useful?

**IZZI vs. IZZI 2**

— List the similarities and the differences.

— Which game do you find harder? Which do you find more interesting? Why?

— How do your problem solving strategies change from one game to the other?

— Can you imagine a new game that blends the two games?

**Jigsaw puzzles** . Compare IZZI or IZZI 2 with conventional jigsaw puzzles.

— List the similarities and the differences.

— Which game do you find harder? Which do you find more interesting? Why?

— How do your problem solving strategies change from one game to the other?

— Can you imagine a new game that blends the two games?

**Apply strategies to other situations** . Applying what you have learned to other situations is one of the best ways to make it your own.

— Mathematics. Discuss connections between IZZI and IZZI 2 puzzles and solving problems in mathematics.

— Decorative arts. Fabrics, wallpaper and floor tiles often use patterns similar to the ones in IZZI and IZZI 2. Make a visually pleasing pattern with a few of the IZZI or IZZI 2 pieces, and develop it into a repeating pattern. You can alter the shapes or colors.

— Combinations. Can you name real-life situations in which it is important to count the number of possible combinations of several elements?

**Classifying puzzles** . These discussion questions further develop a critical understanding of problem solviing skills.

— Compare puzzles. Which puzzles are best for developing which types of problem solving skills?

— Classify puzzles using Venn diagrams. What are the attributes for classifying puzzles? Invent new types of puzzles for categories with no members.